

A Markov Model for Dynamic Behavior of Ranging Errors in Indoor Geolocation Systems

Mohammad Heidari, *Student Member, IEEE*, and Kaveh Pahlavan, *Fellow, IEEE*

Abstract—Recently, considerable attention has been devoted to modeling and analysis of the behavior of the ranging error in indoor environment. The ranging error modeling is essential in design of precise time of arrival (TOA) based indoor geolocation systems. In this paper we present a new framework for simulation of the dynamic spatial variations of ranging error observed by a mobile user based on an application of Markov model. The model relegates the behavior of ranging error into four main categories associated with four states of the Markov process. The parameters of the model are extracted from empirical data collected from a measurement calibrated ray tracing (RT) algorithm in a typical office environment. Results of simulated errors from Markov model and actual errors from empirical data show close agreement.

Index Terms—Channel modeling, indoor geolocation, Markov chain, positioning, and ranging error.

I. INTRODUCTION

RECENTLY, indoor geolocation technology has attracted significant attention, and a number of commercial and military applications are emerging in this field [1]. Indoor channel environments suffer from severe multipath phenomena and blockage of the direct path (DP), creating a need for novel approaches in design and development of systems operating in these environments [2, 3]. Precise indoor geolocation systems are designed based on range measurements from time of arrival (TOA) of the direct path (DP) between transmitter and receiver which is severely challenged by unexpected large errors [4]. Therefore, the ranging error modeling is essential in the design of precise TOA-based indoor geolocation systems.

There are empirical indoor radio propagation channel models available in the literature aiming primarily at telecommunication applications [5, 6]. These models were designed prior to the understanding of the indoor localization problem and hence did not concern the behavior of ranging error in indoor environment. Therefore, they do not provide a close approximation to the empirical observations of the ranging error [7]. More recently, indoor radio propagation channel models designed for ultrawideband (UWB) communications have paid indirect attention to the indoor geolocation problem [8]. However, these indirect models have not paid special attention to occurrence of undetected direct path (UDP) conditions, which are the main cause of large errors in ranging estimates. The first direct empirical model for ranging error is reported in [7, 9]. These new direct models, however, do not

address the spatial correlation of the ranging error behavior observed by a mobile user.

This paper presents a new methodology and framework for modeling and simulation of dynamic spatial variations of ranging error observed by a mobile user based on an application of Markov chain. We categorize the ranging error into four different classes and present clarifications as to the statistical occurrence of each class of ranging error. Then we associate each class of ranging error to a state of a Markov process which can be used for simulation of spatial behavior of ranging error for a mobile user randomly traveling in a building. Models for dynamic behavior of ranging error are essential for the design and performance evaluation of tracking capabilities of the proposed algorithms for indoor localization. The parameters of the model are extracted from the empirical data collected from a measurement calibrated ray tracing (RT) algorithm on the third floor of the Atwater Kent Laboratory (AK Labs) at the Worcester Polytechnic Institute (WPI).

II. DYNAMIC BEHAVIOR OF RANGING ERROR

A. Behavior of Ranging Error

The indoor channel impulse response (CIR) is characterized as

$$h(t) = \sum_{k=1}^{L_p} \alpha_k \delta(t - \tau_k) \quad (1)$$

where L_p represents the number of multipath components, and $\alpha_k = |\alpha_k|e^{j\phi_k}$ and τ_k represent the complex amplitude and TOA of the k^{th} path, respectively [1]. Let $\alpha_{DP} = \alpha_1$ and $\tau_{DP} = \tau_1$ represent the amplitude and TOA of the DP component, respectively. In TOA-based positioning systems the distance between the antenna pair is obtained using $d = \tau_{DP} \times c$, where τ_{DP} and c represent the time of flight and the speed of light, respectively. The range estimate is determined using $\hat{d} = \tau_{FDP} \times c$, where τ_{FDP} represents the TOA of the first detected path (FDP) of the CIR. The distance measurement or ranging error in such systems is then defined as $\varepsilon = \hat{d} - d$ [7, 9]. In the past decade, empirical results from software simulation using RT [7], wideband [3] and UWB [9] measurements of the indoor radio propagation have revealed occurrence of a wide variety of ranging errors. Depending on relative location of the transmitter and receiver, and their position with respect to metallic objects these errors can be divided into four categories [4, 7]; Detected Direct Path (DDP), Natural Undetected Direct Path (NUDP), Shadowed Undetected Direct Path (SUDP), and No Coverage (NC).

For arbitrary locations of transmitter and receiver, the observed ranging error, ε , corresponds to one of the above categories. Figure 1 illustrates the areas associated with the

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The authors are with the Center for Wireless Information Network Studies, ECE Dept., Worcester Polytechnic Institute, 100 Institute Rd., Worcester, MA 01609 (e-mail: {mheidari, kaveh}@wpi.edu).

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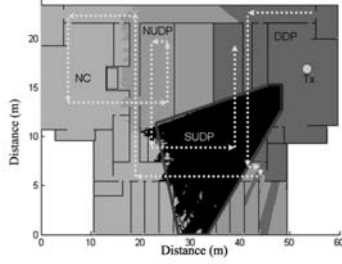


Fig. 1. Receiver location classification for different classes of ranging error in indoor environment

four classes of ranging error in the third floor of the AK Labs at WPI for the specified location of the transmitter. To determine the areas we have used the measurement calibrated RT software previously used in [7] to generate 14000 samples of CIR for different locations of the receiver in the building.

The class of ranging error associated with each receiver location, is defined according to the power of the DP component and the total CIR power given by

$$P_{DP} = 20\log_{10}(|\alpha_{DP}|) \quad (2a)$$

$$P_{tot} = 10\log_{10}\left(\sum_{k=1}^{L_p} |\alpha_k|^2\right) \quad (2b)$$

In DDP conditions, DP is available and $P_{DP} > \eta$ where η is defined as the detection threshold of the process and is determined using existing pathloss models [7]. In DDP class of receiver locations the observed values of ranging error are small, which are mainly multipath induced [7]. When $P_{DP} < \eta$ the receiver location experiences a UDP condition. Further we sub-categorize this condition into NUDP and SUDP. Specifically, the latter is distinguished by considering the physical layout of the building where the DP component is blocked by a metallic object which results in unexpected large ranging error. However, NUDP is described by blockage of DP but to non-metallic objects which results in relatively smaller ranging errors [4, 10]. Finally, $P_{tot} < \eta$ results in NC conditions in which the received power is not sufficient for establishing the communication link between antenna pair [7] and ranging estimate is unavailable.

B. Four Ranging States of the Markov Model

As mobile client randomly travels in the building, as shown in Fig. 1, and depending on the region of movement it experiences different classes of ranging errors. Using the four classes of ranging error observed in indoor environment, we can construct a four-state first-degree Markov model to represent the dynamic behavior of ranging error. Random movement of the mobile user results in change of its observed class of ranging error, with particular probabilities which in general are function of detection threshold, speed of movement, type of movement, and geometry of the building. The general Markov model representation for indoor positioning is described in Fig. 2, where it can be concluded that transition from DDP state to NC state is only possible through one of the UDP states.

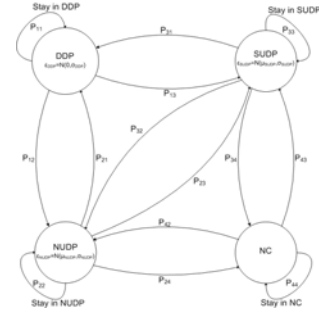


Fig. 2. Presented four-state Markov model for indoor environment

Following the methodology described in [10,11], transition probabilities are defined as the rate of switching between Markov states or staying in the same state, accordingly

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{bmatrix} \quad (3)$$

where p_{ij} s are defined as

$$p_{ij} = \begin{cases} p\{y_i = \xi_i | y_{i-1} = \xi_i\} & i = j \\ p\{y_i = \xi_j | y_{i-1} = \xi_i\} & i \neq j \end{cases} \quad (4)$$

in which y_i and $\xi_i \in \{DDP, NUDP, SUDP, NC\}$ represent the current state of the ranging error and the Markov state associated with the current state, respectively. The eigenvector associated with the eigenvalue $\lambda_1 = 1$ of P determines the expected average waiting time in each state which is used in initialization of the Markov chain.

Next, assuming homogeneous transition probabilities in continuous time, i.e. $P[X(s+t) = j | X(s) = j] = P[X(t) = j | X(0) = i] = p_{ij}(T_n)$, we convert the discrete Markov chain in Eq. (3) to an equivalent continuous time Markov chain to extract the staying time distributions of each state. The staying time in a certain state can then be described by an exponential random variable $P[T_i > t] = e^{-v_i t}$. Similar to methodology described in [11], v_i s can be determined by solving the respective Chapman-Kolmogorov equation.

$$[v_1 \ v_2 \ v_3 \ v_4] = \begin{bmatrix} \frac{1}{1-p_{11}} & \frac{1}{1-p_{22}} & \frac{1}{1-p_{33}} & \frac{1}{1-p_{44}} \end{bmatrix} \quad (5)$$

Determining exponential parameters allows us to simulate the average wait time in each state and to compare them with results of the empirical data.

C. Simulation Parameters using Empirical Data

To complete the dynamic Markov model, the statistics of the ranging error in different regions of a building and transitions probability matrix defined in Eq. 3 are required.

1) *Markov model parameters:* We generated 14000 CIRs for the different receiver locations in the building and categorized them into DDP, NUDP, SUDP, and NC classes, according to Eq. (2) and the physical blocking condition. We specifically used $\eta = -80 \text{ dBm}$ as the detection threshold. Once the class of each receiver location was identified, we developed

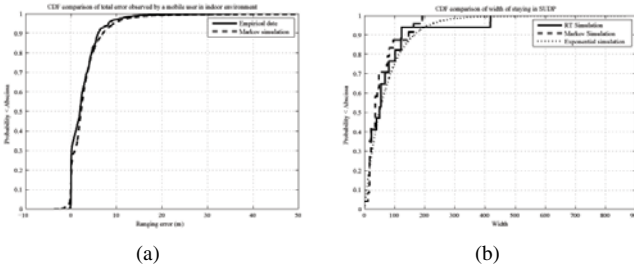


Fig. 3. Dynamic behavior of ranging error in indoor environment, a) Comprehensive ranging error through different states which illustrates the underlying ranging error distributions in each state and Markov model for spatial correlation of ranging error states are applicable and b) CDF comparison of length of staying in SUDP class of receiver location for empirical data, Markov simulation, and Exponential modeling of the staying times

TABLE I
STATISTICS OF RANGING ERRORS FOR DIFFERENT STATES

Class \ Statistics	$\mu(m)$	$\sigma(m)$
DDP	0.01	0.12
NUDP	0.12	0.27
SUDP	4.12	1.63

a *random walk* process that simulated the mobile terminal random movement amongst 14000 receiver locations.

The set up of the *random walk* was designed to emulate random movements of a real mobile client in indoor environment by walking in hallways, offices, and labs. The process resulted in 7000 CIRs associated with the receiver locations of the *random walk* which are used to calculate the transition probability matrix of the Markov model using (3) and (4)

$$P = \begin{bmatrix} 0.9645 & 0.0085 & 0.027 & 0 \\ 0.0156 & 0.9401 & 0.0156 & 0.0287 \\ 0.1259 & 0.0979 & 0.7622 & 0.014 \\ 0 & 0.0208 & 0.0469 & 0.9323 \end{bmatrix} \quad (6)$$

Note that $p_{14} = p_{41} = 0$ confirms the absence of the link between DDP and NC classes. The corresponding waiting times vector is found by (5) to be

$$v = [28.16 \quad 16.69 \quad 4.20 \quad 14.77] \quad (7)$$

2) *Ranging error statistics*: Empirical models using measurement and RT have confirmed that ranging errors occurring in each state can be modeled with normal distribution whose moments are functions of bandwidth of the channel [9, 10]. Table I summarizes the statistics of ranging error for different states of the presented Markov model for a bandwidth of 500 MHz obtained from the analysis of CIRs of the same 14000 receiver locations in the third floor of the AK Labs.

Using the statistics of ranging error and the parameters of the Markov model of Fig. 2 provided by Eq. (6) enable us to simulate the dynamic behavior of ranging error observed by the mobile user. In order to validate the effectiveness of the Markov model we initialized and ran the Markov process with the transition probabilities defined in Eq. (6). According to the class of ranging error produced by Markov model, we

simulated the ranging error according to the statistics given in Table I. Figure 3.a shows the CDFs of ranging errors observed by a mobile user obtained from empirical data and the Markov model. The closeness of the results illustrates the validity of the model to generate similar statistics for the ranging error. Comparison of empirical and simulated ranging error in individual classes of receiver location also showed close agreement. Another measure to illustrate the dynamic behavior is the length of stay in a given class of receiver location. Figure 3.b illustrates the CDFs of the length of stay in SUDP class of receiver locations for a random mobile client. The empirical results from the simulation of *random walk* by RT and results obtained from Markov modeling show close agreement and as expected from a Markov model both of them follow an exponential distribution. Analyzing the spatial correlation of ranging errors in a certain class of receiver location is essential to fully model the dynamic behavior of ranging error in indoor environment. Its treatment, however, is beyond the scope of this letter.

III. CONCLUSION

In this letter we presented a novel application of Markov model for modeling the dynamic behavior of the ranging error in a typical indoor geolocation application. The parameters of the model were extracted from empirical data obtained from 14000 CIRs in the third floor of the AK Labs at WPI. The result of simulation for the ranging error using the Markov model was shown to provide close agreement with the results of empirical data in terms of statistics of ranging error and length of stay in a certain class of receiver location.

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